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Influence of turbulence intensity on particle drag coefficients

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Abstract

The effect of turbulent eddies on the motion of the dispersed phase is often ignored by assuming a standard drag coefficient as applied to quiescent flow. Such an assumption can lead to large errors when predicting the dispersed phase concentration profile of an industrial flow under turbulent conditions. Recently, [G.L. Lane, M.P. Schwarz, G.M. Evans, Numerical modelling of gas-liquid flow in stirred tanks, Chem. Eng. Sci. 60 (2005) 2203–2214] developed a model relating the drag coefficient to fluid turbulence characteristics through the dimensionless group Stokes number. Their model has been successfully applied to predict gas holdups of a mechanically stirred tank over the range of $St < 0.7$.

The present study focused on broadening the analysis by Lane et al. Specifically, the aim was to examine the effects of dispersed phase density and size on the applied drag force under turbulent conditions and to extend the much needed experimental data on particle drag coefficients in free-stream turbulence, as a function of solid particles characteristics. This was achieved by a systematic experimental approach using particles of different sizes and densities in two distinct turbulent flow fields generated by oscillating grids. The results indicated that the reduction in settling velocity is a function of both particle size and density and turbulent characteristics with maximum interaction between the continuous and dispersed phases occurring at low ratios of particle density to liquid density and high turbulence intensities. Richardson number, a dimensionless group, was employed to capture the effects of these parameters on the drag coefficient.

Keywords: Drag coefficient; Isotropic turbulence; Particle settling velocity

1. Introduction

Multiphase flows involving suspensions of (solid, liquid and gaseous) particles in liquids are frequently encountered in many industrial processes, such as leaching, flotation, polymerization and crystallization [1]. Typically, these multiphase operations are carried out under turbulent conditions of varying intensity in processing vessels, such as bubble columns, loop reactors and mechanically agitated vessels. In these processes often a uniform dispersion of particles is achieved due to the interaction between turbulent eddies and the dispersed phase. A better understanding of such interaction is fundamental to the effective design, modelling and operation of multiphase systems.

In simulations of multiphase processes using computational fluid dynamics (CFD), the interaction between continuous and dispersed phases is usually taken into account in the correlation for the particle drag coefficient. The majority of (standard) drag coefficient correlations is developed based on the motion of particles in stagnant fluid. While the standard relationships are successfully employed to model the motion of particles in laminar flow, their use for simulation of particle motion under free-stream turbulence conditions (i.e. turbulence generated by external sources) may result in large errors, especially when predicting the dispersed phase concentration profile [2].

As the literature indicates, the modification of the drag coefficient by turbulence has been a subject of investigations for many years. The studies carried out by Schwartzberg and Treybal [3] and Nouri and Whitelaw [4] indicated a significant reduction in particle settling velocities, with the velocity measurements being as low as 30% of those in a stagnant liquid. That in turn implies an increase in drag coefficients under free-stream turbulence conditions. Their experimental results, however, suffered from a degree of uncertainty since the slip velocity was obtained as a small difference between the particle and liquid velocities, which are both typically much larger than the slip velocity, and hence the measured slip velocity was subjected to a large error. In a later study, Brucato et al. [1] reported settling velocities as low as 15% of the particle terminal velocity in a quiescent liq-

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Nomenclature

C_A	added mass coefficient
$\overline{C_\mathrm{D}}$	effective drag coefficient in turbulence
$C_\mathrm{D,0}$	particle drag coefficient in stagnant liquid
d	particle diameter (m)
L	turbulent integral length scale (m)
Ri	Richardson number
St	stokes number
T_L	turbulent integral time scale (s)
u_0	r.m.s. velocity of turbulence (m s^{-1})
U_S	Slip velocity (m s^{-1})
U_T	particle terminal velocity (m s^{-1})
Greek letters	
λ	Kolmogoroff length scale (m)
ρ	fluid density (kg m^{-3})
ρ_P	particle density (kg m^{-3})
τ_P	particle relaxation time (s)

uid, while performing experiments using a Taylor–Couette type vessel. Brucato's experimental data were found to be consistent with those of Magelli et al. [5] when plotted as $U_\mathrm{S}/U_\mathrm{T}$ versus λ/d, where U_s is the slip velocity, U_T is the terminal velocity, and λ is the Kolmogoroff scale of turbulence. A correlation for the ratio of the effective drag coefficient ensemble-averaged over turbulent fluctuations to that of a single particle settling through a stagnant liquid, $\overline{C_\mathrm{D}}/C_\mathrm{D,0}$, was proposed by Brucato et al. [1][1]:

$$\overline{C_\mathrm{D}}/C_{D,\mathrm{o}} = 1 + 8.76 \times 10^{-4}(d/\lambda)^3 \tag{1}$$

Although, the above correlation provides a good description of the experimental data of both Brucato et al. [1] and Magelli et al. [5], care must be taken when applied to particles with different densities as the correlation does not provide allowances for changes in particle density.

More recently, Lane et al. [2] have proposed a drag coefficient correlation using available literature data. In the model, the effect of turbulence on the drag coefficient was correlated in terms of a dimensionless group known as Stokes number:

$$U_\mathrm{S}/U_\mathrm{T} = 1 - 1.4St^{0.7}\exp(-0.6St) \tag{2}$$

The Stokes number, *St*, is a measure of the time taken for a particle to respond to an interacting turbulent eddy, and hence defined as:

$$St = \frac{\tau_\mathrm{P}}{T_\mathrm{L}} \tag{3}$$

where τ_P is the particle relaxation time and T_L is the turbulent integral time scale. In an isotropic turbulence, in which the flow characteristics are statistically independent of direction, the integral time scale can be defined as: $T_\mathrm{L} = L/u_0$ where L is the integral length scale and u_0 is the r.m.s. velocity of turbulence. The relaxation time of a particle can be calculated according to Bel F'dhila and Simonin [6]:

$$\tau_\mathrm{P} = \frac{\rho_\mathrm{P}/\rho + C_\mathrm{A}}{(3/4)(C_\mathrm{D}/d)U_\mathrm{T}} \tag{4}$$

where C_D is the drag coefficient in stagnant liquid, ρ_P is the particle density, ρ is the fluid density, d is the particle size and $C_\mathrm{A} = 0.5$ is the added mass coefficient [7].

The model by Lane et al. [2] captured the continual decrease in settling velocity of both solid and gaseous particles as the Stokes number increased, providing especially a good fit to data for bubbles. This model has been successfully applied to simulate the gas bubble distribution within a stirred tank [2]. However, the model application over the range of $St > 0.7$ is subjected to experimental verification. In the absence of experimental data, the authors suggested the following explanations on how the relationship extrapolates to higher values of Stokes number when either the particle has a large relaxation time or the time scale of turbulence is much shorter than that of the particle. Since, in either case, the effect of turbulence is considered to be negligible (i.e. as $\tau_\mathrm{P}/T_\mathrm{L} \to \infty$, $U_\mathrm{S}/U_\mathrm{T}$ ratio reaches unity), it was suggested that the plot of $U_\mathrm{S}/U_\mathrm{T}$ versus Stokes number must have a minimum where the maximum interaction between turbulence and particles occurs.

The experimental investigations on turbulence have been conducted in either a decaying homogeneous turbulence with a mean flow, such as water tunnels, or in a stationary homogeneous turbulence field with no mean flow, such as those generated by oscillating grids [8]. The former generators, however, have limited application range especially since increasing the fluid velocity is the only means to increase the intensity of the turbulence. Further, the particle settling velocity measurements can be influenced by the mean flow, making accurate determination of the velocities very difficult [9]. The stationary turbulence generators are hence often the preferred option as the measurement errors are minimised in the absence of a mean flow.

The review of the work to date on how turbulence influences the drag on bubbles and particles, has highlighted two main points. Firstly, there is a clear advantage of using a stationary turbulence generator to eliminate any net mean flow effects. Secondly, the dimensionless drag-turbulence relationship proposed by Lane et al. [2] requires experimental confirmation at higher Stokes numbers. The aim of this study, therefore, was to extend the much needed experimental data on particle drag coefficients in free-stream turbulence, as a function of solid particles characteristics (including size, density and terminal velocity), and to compare the data with the proposed model by Lane et al. [2].

2. Experimental methods

In the present study, an oscillating turbulence generator similar to that of Yang and Shy [9] was employed to produce stationary near isotropic turbulent flow fields. In such devices, the oscillating grids generate a system of wakes and jets that merge with each other to produce a sustained turbulence with zero mean flow [10]. The turbulent intensity can be varied sim-

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[1] Under steady-state conditions, the slip velocity and drag coefficient are related through a simple force balance between the buoyancy force and drag force: $\overline{C_\mathrm{D}}/C_\mathrm{D,0} = 1/(U_\mathrm{S}/U_\mathrm{T})^2$.

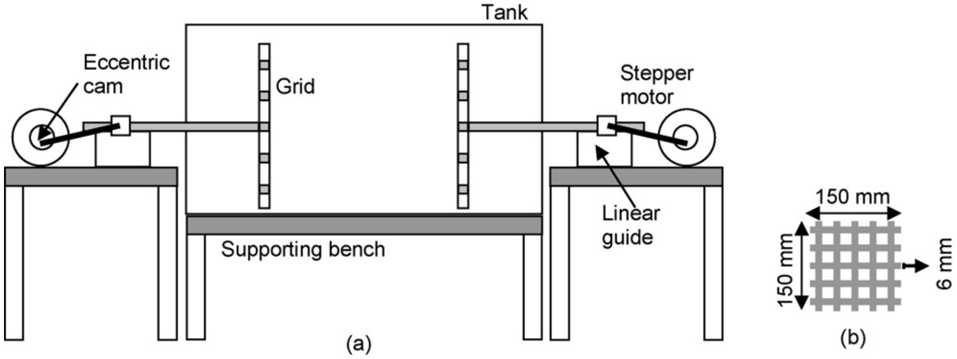

Fig. 1. (a) A schematic representation of the turbulent generator, (b) the grid geometry.

Table 1
Turbulent flow characteristics [10]

Grid frequency (Hz)	r.m.s. turbulent velocity (mm/s)	Integral length scale (mm)
3	7.2	3
4	9.6	3
5	12.0	3
6	14.4	3
7	16.8	3
8	19.2	3

ply by changing the vibration frequency, the size of the strokes and/or the separation distance between the grids. A relationship between these operational parameters and the characteristics of the resultant turbulent field has been already established [8].

A schematic representation of the generator used here is shown in Fig. 1a. The device consisted of a rectangular perspex tank with a width of 300 mm containing a pair of vertically oriented grids of size 150 mm × 150 mm. The grids were made of 6 mm thick aluminium sheets with a mesh size of 30 mm (i.e. the distance between centres of two successive openings was 30 mm) and an overall opening of 64% (Fig. 1b). Perfect horizontal and vertical alignments of the square-openings were achieved using a high precision laser cutting technique. The abovementioned design specifications were set according to the literature to prevent formation of unstable flow structures [9].

Connecting rods supported by linear bearings were then used to connect the grids to stepper motors through eccentric cams. To minimise the possibility of transmitting vibration from motors to the flow field, the tank was installed on a separate supporting bench [11].

Generally, the system was designed to operate at different frequencies and strokes of up to 10 Hz and 50 mm, respectively. Design considerations allowing for adjustment of the separation distance between the grids were also made. For the purpose of this study however, the stroke was fixed at 20 mm while the mean separation distance between the grids was set at 110 mm. The motors were setup so that the grids moved in-and-out together, rather than side-to-side. The flow characteristics of turbulence under these conditions for particular frequencies have already been determined by Yang and Shy [9] for a similar system. Since the same design principles as those of Yang and Shy [9] are used to fabricate our turbulent generator, it is reasonable to assume that identical flow fields are formed under the same operational conditions. Data analysis was hence performed according to this assumption. Yang and Shy [9] measured the mean fluctuation velocity of the turbulence using a Laser Doppler Velocimetry (LDV) system while the length scale of the turbulence was estimated to be about 3 mm for frequencies varying from 3 to 8 Hz. A summary of the flow characteristics is presented in Table 1.

The experimental studies were carried out using precision spherical particles made of Nylon and Teflon. The physical characteristics of the particles are given in Table 2. The particle Reynolds numbers presented in Table 2 were calculated using the particle terminal velocity in a quiescent liquid, U_{T}, which was obtained experimentally.

A systematic approach was adopted in the experimental work. First, the operational parameters including the vibration frequency, the size of the stroke and the distance between the grids were set. A turbulent flow field was then established by out-of-phase oscillation of the grids. A test particle was carefully released at the top section of the vessel while maintaining a zero velocity at the time of the discharge. The particle descent was captured using a high speed video camera. Phantom5 software was used to analyse the data obtained in the near isotropic turbulent region, which has a width of approximately 40 mm and height of about 100 mm located in the centre between the grids [9]. The settling velocity for each particle was determined from a plot of vertical distance versus time (i.e. the particle trajectory in y direction as a function of time). Each run was repeated a number of times with the velocity being reported as the average value. In the present study, the particle slip velocity is equal to

Table 2
Physical characteristics of particles

Particle	Particle size (mm)	Specific gravity	Particle Reynolds number
Nylon	2.38	1.14	183
	3.18	1.14	325
	4.76	1.14	605
	7.94	1.14	1299
Teflon	2.38	2.30	662
	3.18	2.30	986
	4.76	2.30	1869
	7.94	2.30	4108

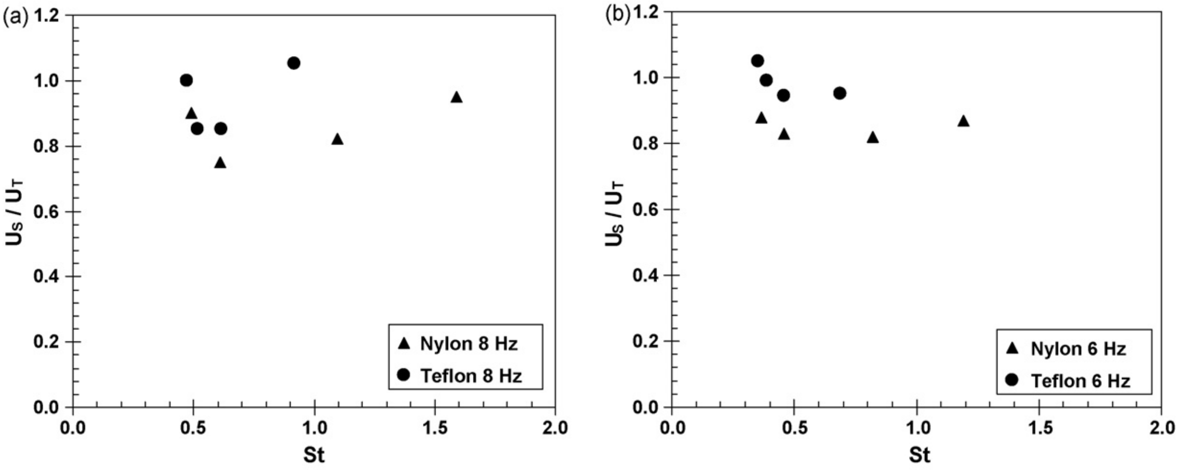

Fig. 2. Effect of turbulence on the settling velocity of Nylon and Teflon particles generated at oscillating frequencies of (a) 8 Hz and (b) 6 Hz which corresponded to r.m.s. turbulence velocities of 19.2 and 14.4 mm/s, respectively.

its settling velocity since the liquid mean velocity relative to the wall is zero.

3. Results and discussion

The motion of different size particles made of Nylon and Teflon were examined in two different turbulent flow fields generated by oscillating grids as outlined in the experimental section. The results of these studies are presented in Figs. 2–5. The experimental uncertainty was generally less than 5% of the particle terminal velocity value.

Fig. 2a and b illustrate plots of velocity ratio, denoted by U_S/U_T, as a function of Stokes number for frequencies of 8 and 6 Hz, respectively. It is generally understood that free-stream turbulence in a liquid media may lead to a significant reduction in settling velocity of a solid particle. The reduction is considered to be a function of turbulence and particle characteristics including particle size and density. For the case shown in Fig. 2a under set turbulent conditions the reduction in velocity ratio for Nylon particles is as high as 25%.

As can be seen from Fig. 2a, generally an increase in Stokes number, through increasing the particle diameter, reduced the U_S/U_T ratio. The velocity ratio, however, reached a minimum value at a particular Stokes number for which the effect of turbulence was at its maximum. At higher Stokes numbers, where the particle diameter is much greater than the integral length scale of turbulence, the velocity ratio increased returning to values around unity for both Nylon and Teflon particles. This is because, when $d/L \gg 1$, there are no turbulent eddies of sufficient size or energy to deflect the particle from its path.

Similar behaviour was exhibited by the Nylon and Teflon particles at lower turbulence intensities as shown in Fig. 2b for the case of 6 Hz frequency. However, the increase in the velocity ratio beyond the minimum value of U_S/U_T (i.e. $U_S/U_{T,\min}$) is not as steep as that observed in Fig. 2a for the 8 Hz frequency. Indeed, for the range of Stokes numbers studied, the velocity ratios corresponding to Nylon and Teflon particles do not return to values around one. Consequently, shapes of the plots appear to be different when one compares Fig. 2a and b. Clearly, the turbulence intensity plays a key role in defining the shape of the

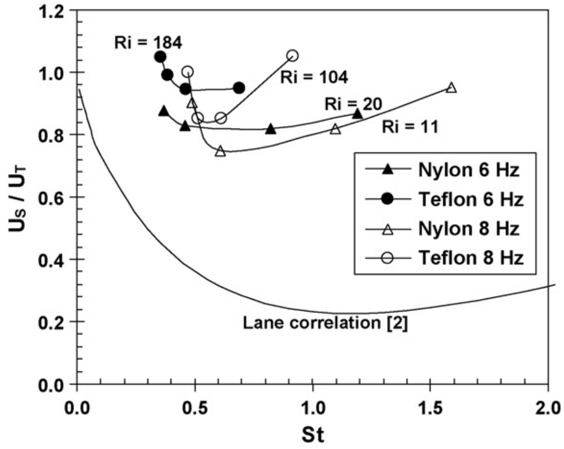

Fig. 3. Comparing the experimental data for settling velocity of solid particles in turbulent flows with the model predictions of Lane et al. [2]. The results are presented as a function of Stokes and Richardson numbers.

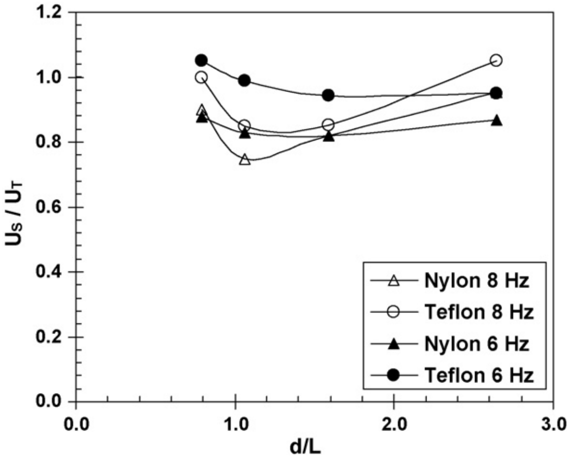

Fig. 4. The effect of particle size relative to the integral length scale of turbulence on the particle settling velocity.

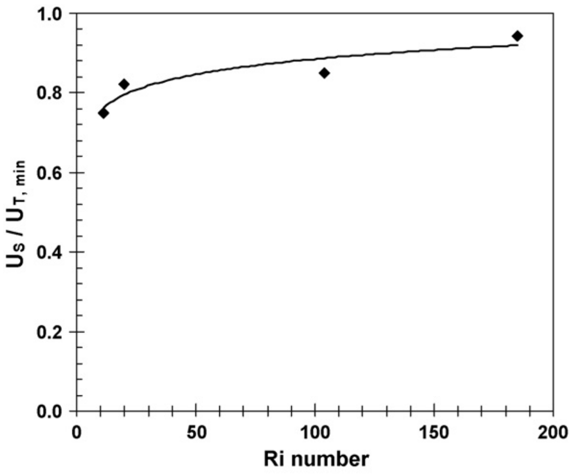

Fig. 5. The maximum interaction between solid particles and turbulent eddies as a function of Richardson number.

curve with steeper slopes being observed at the higher turbulence intensity associated with the 8 Hz oscillation frequency.

The analysis of multiphase turbulent flows involving bubbles by Lane et al. [2] suggests that there is a unique correlation between the velocity ratio and Stokes number allowing all relevant experimental data to collapse onto a single curve. This plot is reported to have a minimum at $St = 1$. The correlation has been verified using limited literature data for $St < 0.7$ with particle sizes being smaller than the integral length scale (i.e. $d/L < 0.3$).

However, as is evident in Fig. 3, there seems to be a considerable discrepancy between Lane's correlation [2] and the experimental data obtained in this study. Here, the data were collected over the range of $0.35 < St < 1.6$ employing test particles of size 2–8 mm in a turbulent flow field with integral length scale of 3 mm (i.e. $0.6 < d/L < 2.7$) and velocity ratios of $u_0/U_T < 0.3$, where Lane's correlation [2] has been developed based on experimental data collected over $St < 0.3$ for small d/L and large u_0/U_T ratios (i.e. $d/L < 0.3$ and $u_0/U_T > 1$). Under the former conditions, a minimum interaction between dispersed and continuous phases is expected as the turbulent eddies lack sufficient size or energy to deflect the particle from its path and hence lead to a significant reduction in the particle settling velocity. That, in turn, could explain the observed discrepancy between the data collected in the present study and the correlation by Lane et al. [2]. The presented results hence suggests that besides St number, both d/L and u_0/U_T ratios need to be considered as separate entities in developing a correlation for particle drag coefficient in turbulent flows.

In addition, the relationship between the velocity ratio and Stokes number for solid particles does not appear to be unique for the range of experimental conditions studied in this project. Instead, a family of curves were obtained, to the suggestion by Lane et al. [2], that result for both bubbles and particles could be fitted by a single curve. The work presented here, with the additional experimental data, suggests that a family of curves is required, and that the original correlation by Lane et al. [2] represents just one of those curves. While our experimental data are not enough to clearly demonstrate the existence of a family of distinct curves in a plot of U_S/U_T versus St number, they clearly do not support the existence of a unique relationship as proposed by Lane et al. [2]. However, it is our intention to carry out a series of experiments in near future to establish the exact relationship between the particle slip velocity and parameters including St number, d/L and u_0/U_T ratios.

In Fig. 4, U_S/U_T has been plotted as a function of d/L. It can be seen that the velocity curve starts to flatten as d/L approaches 1. This behaviour was found to be irrespective of the particle density or turbulence intensity. Moreover, it seems that a minimum value, $(U_S/U_T)_{,\min}$, for U_S/U_T occurred at around $d/L = 1$ for all experimental runs. The most likely reason for this is that maximum interaction between the particle and the turbulent eddies occurs when the particle size is approximately the same as the turbulence integral length scale.

Fig. 4 also shows that the numerical value of $(U_S/U_T)_{,\min}$ is a function of both the particle/liquid density ratio and turbulence intensity. This suggests that the maximum reduction in settling velocity is a function of the inertial forces, due to turbulence, and the net effective weight of the particle, due to gravity. The ratio of these forces is known as the Richardson number, and is defined as:

$$Ri = \frac{g|\rho_P - \rho|}{\rho} \frac{L}{u_0^2} \tag{5}$$

Generally, high values of Ri number are obtained when the force of gravity is dominant (i.e. at high ratios of $|\rho_P - \rho|/\rho$). A low value of Ri number reflects the dominance of turbulence, commonly achieved at high turbulence intensities or low integral length scales. Fig. 4 shows Richardson number plotted as a function of the minimum U_S/U_T ratio, $(U_S/U_T)_{,\min}$, for each of the curves given in Fig. 3. It can be seen that as Ri was increased the effect of turbulence on the settling velocity of particles was reduced to negligible levels, i.e. $(U_S/U_T)_{,\min} \to 1$. At $Ri = 184$, the reduction in $(U_S/U_T)_{,\min}$ was only about 5 percent. Conversely, there appears to be a major effect on $(U_S/U_T)_{,\min}$ below Ri of approximately 20.

4. Conclusions

Solid particles of different sizes and densities were used to investigate the interaction between the continuous and dispersed phases for values of the Stokes number ranging from 0.35 to 1.6. As the Stokes number was increased, through an increase in particle diameter, the U_S/U_T ratio decreased reaching a minimum value at d/L of about 1. At this value, there is maximum interaction between the turbulence and the particle. The U_S/U_T ratio began to increase as the Stokes number was further increased returning to a value of 1 at d/L ratios of more than 2.5. This trend was similar to that proposed by Lane et al. [2], except that a family of curves was obtained rather than a single unifying curve. Particle density and turbulence intensity were found to be the key parameters affecting the settling velocity, and from this the Richardson number was used to correlate the minimum reduction in the velocity ratio, $(U_S/U_T)_{,\min}$. Also, negligible reduction in the settling velocity was observed for Ri values greater than ∼200.

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